

Modeling Linear Situations

Creating a Table and Function



A 747 airliner has an initial climb rate of 1800 feet per minute until it reaches a height of 10,000 feet.

1. Identify the independent and dependent quantities in this problem situation.
Explain your reasoning.

I: time in minutes
D: height in feet

* the rate allows us to identify which quantities are being compared.

2. Describe the units of measure for:
 - a. the independent quantity (the input values).

minutes.

- b. the dependent quantity (the output values).

feet.



3. Which function family do you think best represents this situation? Explain your reasoning.

Linear.

The constant rate of increase will create a straight line when graphed.



5. Write the independent and dependent quantities and their units of measure in the table. Then, calculate the dependent quantity values for each of the independent quantity values given.

Although it is a convention to place the independent quantity on the left side of the table, it really doesn't matter.



	Independent Quantity	Dependent Quantity
Quantity	time	height
Units	min.	feet
	0	0
	1	1800
	2	3600
	2.5	4500
	3	5400
	3.5	6300
	5	9,000
Expression	t	$1800t$

2

Why do you think it was chosen as the variable?



6. Write a function, $h(t)$, to describe the plane's height over time, t .

$$h(t) = 1800t$$

Curriculum Learning



Calculate the Rate of Change between the following ordered pairs: $\frac{\Delta h}{\Delta t}$

a. (2.5, 4500) and (3, 5400)

$$= \frac{5400 - 4500}{3 - 2.5} = \frac{900}{.5} = 1800$$

b. (3, 5400) and (5, 9000)

$$= \frac{9000 - 5400}{5 - 3} = \frac{3600}{2} = 1800$$

c. (2.5, 4500) and (5, 9000)

$$= \frac{9000 - 4500}{5 - 2.5} = \frac{4500}{2.5} = 1800$$

Remember, if you have two ordered pairs, the rate of change is the difference between the output values over the difference between the input values.



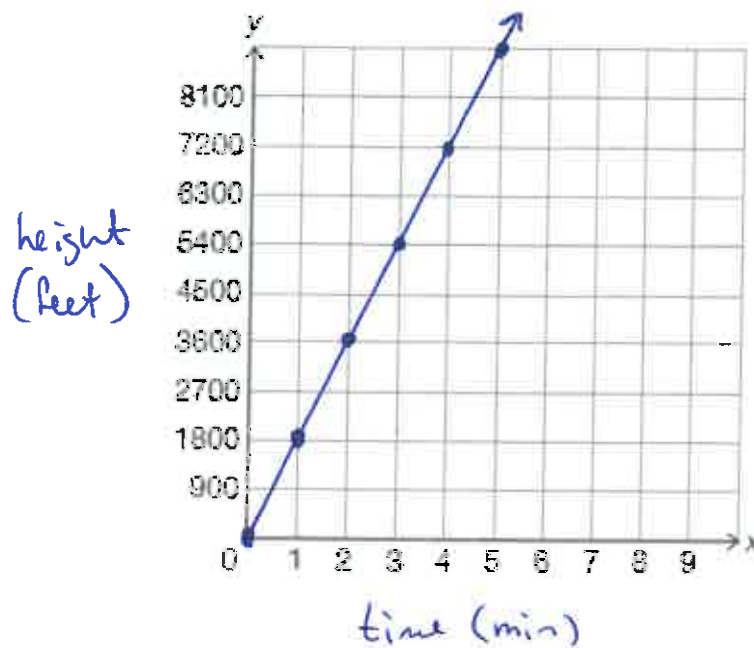
What do you notice about the rates of change?

they are all = to 1800 no matter what 2 points are chosen.

Making a Graph



4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.



- a. What is the slope of this graph? Explain how you know.

Slope = 1800.

the function $h(t) = 1800t$ is in $y = mx + b$ form, where m is the slope.

So here, $m = 1800$.

- b. What is the x-intercept of this graph? What is the y-intercept? Explain how you determined each intercept.

x-intercept = 0 ← the graph crossed the x-axis at 0.

y-intercept = 0 ← the graph crossed the y-axis at 0.



- c. What do the x- and y-intercepts mean in terms of this problem situation?

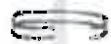
x-intercept means time = 0. that's when the plane takes off.

y-intercept means height = 0. that's means the plane starts on the ground.

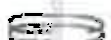
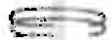
Calculating an Output Value



Let's consider how to determine the height of the plane, given a time in minutes, using function notation.



To determine the height of the plane at 2 minutes using your function, substitute 2 for t every time you see it. Then, simplify the function.



$$h(t) = 1800t$$

Substitute 2 for t . \longrightarrow $h(2) = 1800(2)$

Input value
 $h(2) = 3600$

Two minutes after takeoff, the plane is at 3600 feet.



6. Use your function to determine the height of the plane at each given time in minutes. Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) = \frac{1800(3)}{1} = 5400$

3 minutes after take off
the height is 5400 ft.

b. $h(3.75) = \frac{1800(3.75)}{1} = 6,750$

the height after 3.75 minutes
is 6,750 ft.



c. $h(5.1) = \frac{5.1(1800)}{1} = 9180$

the height after 5.1
minutes is 9,180 ft.

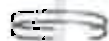
d. $h(-4) = \frac{-4(1800)}{1} = -7,200$

height can't be negative,
so this does not apply.

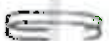
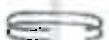
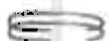
Calculating an Input Value



Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.



To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.



Substitute
7200 for $h(t)$.
output
value.

$$h(t) = 1800t$$

$$7200 = 1800t$$

$$\frac{7200}{1800} = \frac{1800t}{1800}$$

$$4 = t$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.



2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

a. 5400 feet

$$\frac{5400}{1800} = \frac{1800t}{1800}$$

$$t = 3 \text{ minutes.}$$

It takes 3 minutes to reach 5400 ft.

c. 3150 feet

$$3150 = 1800t$$

$$t = \frac{3150}{1800} = 1.75$$

It takes 1.75 minutes to reach 3150 ft.

b. 9000 feet

$$9000 = 1800t$$

$$t = \frac{9000}{1800} = 5$$

It takes 5 minutes to reach 9000 ft.

d. 4500 feet

$$4500 = 1800t$$

$$t = \frac{4500}{1800} = 2.5$$

It takes 2.5 minutes to reach 4500 ft.



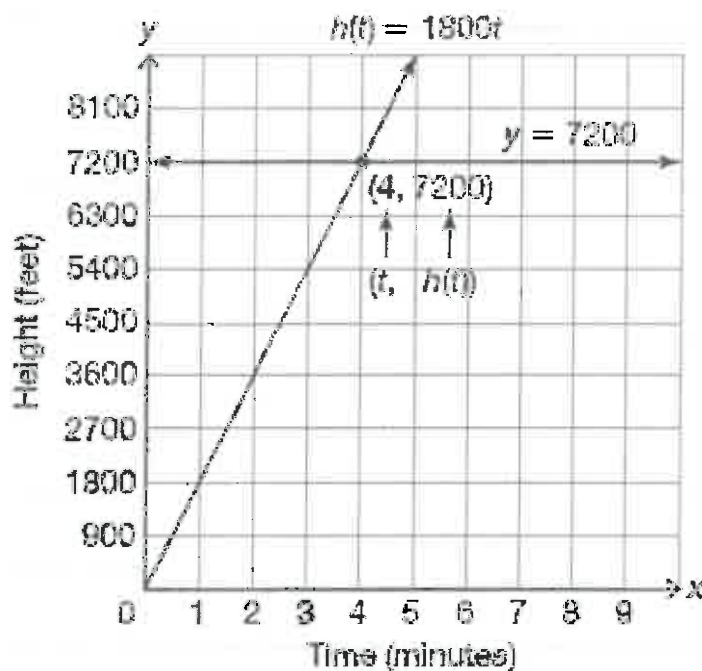
Using a Graph to Find Solutions



You can also use the graph to determine the number of minutes the plane has been flying (input value) given a height in feet (output value). Remember, the **solution** of a linear equation is any value that makes the open sentence true. If you are given a graph of a function, a solution is any point on that graph. The graph of any function, f , is the graph of the equation $y = f(x)$. If you have intersecting graphs, a solution is the ordered pair that satisfies both functions at the same time, or the **intersection point** of the graphs.

To determine how many minutes it takes for the plane to reach 7200 feet using your graph, you need to determine the intersection of the two graphs represented by the equation $7200 = 1800t$.

First, graph each side of the equation and then determine the intersection point of the two graphs.

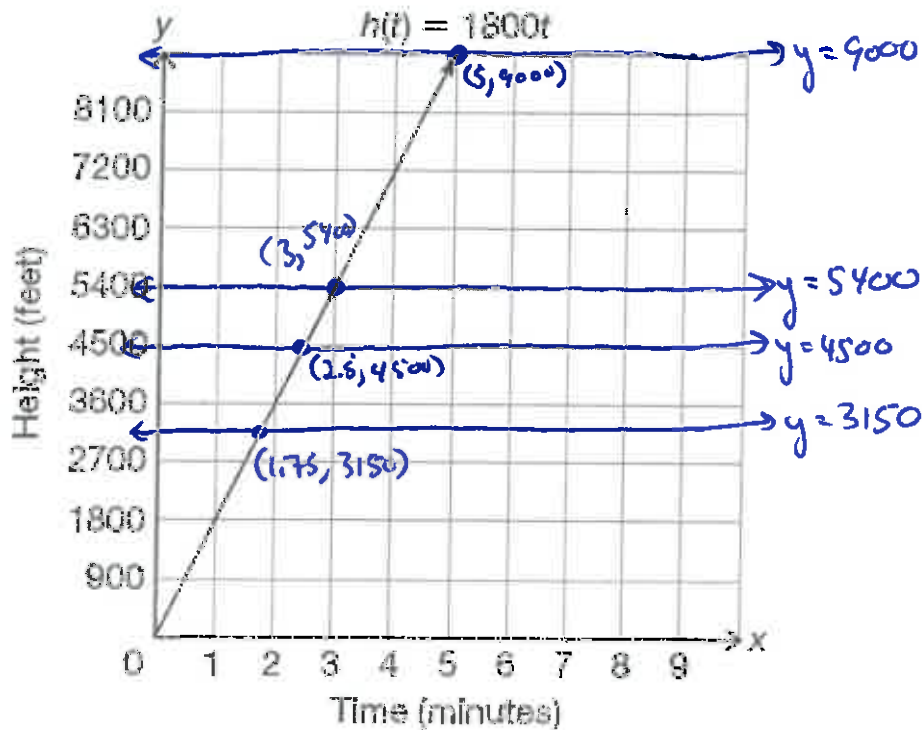


$$\begin{array}{rcl} h(t) = 1800t & & \\ 7200 = 1800t & & \\ \downarrow & & \downarrow \\ y = 7200 & & y = 1800x \\ \text{Solution: } (4, 7200) & & \end{array}$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.



5. Use the graph to determine how many minutes it will take the plane to reach each height.
- $h(t) = 5400$
 - $h(t) = 9000$
 - $h(t) = 3150$
 - $h(t) = 4500$



Talk the Talk



You just worked with different representations of a linear function.

- Describe how a linear function is represented:
 - in a table, when the rate of change between any two points is constant.
 - in a graph, as a straight line
 - in an equation, as $y = mx + b$
- Explain the connection between the form of the function $h(t) = 1800t$ and the equation $y = 1800x$ in terms of the independent and dependent quantities.

x and t are independent quantities.
 y and $h(t)$ are dependent quantities

